APPENDIX – Common Expressions

The number of combinations of *n* distinct objects taken *r* at a time is (from *ref. [9]*, pg. 23):

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}, \quad \text{where} \quad x! = x*(x-1)*(x-2)*...*(x-(x-1)) \tag{A1}$$

If A and B are any two events (*ref. [9]*, pg. 28):

(A2)

Probability{A OR B} = Probability{A} + Probability{B} - Probability{A AND B}

BERNOULLI TRIALS:

A Bernoulli trial can result in success with probability *p* and a failure with probability *q = 1-p*. The probability distribution of the random variable, *X*, representing the number of successes in *n* independent trials is given by the Binomial Distribution (from *ref. [9]*, pg. 117):

$$b(x;n,p) = \binom{n}{x} p^x q^{n-x}, \quad x = 0,1,2,...,n \tag{A3}$$

The probability of *at least r* successes in *n* independent trials is (from *ref. [9]*, pg. 118):

$$1 - P(\#\,successes < r) = 1 - \sum_{x=0}^{r-1} b(x;n,p) \tag{A4}$$

SUMMATION EXPRESSIONS:

$$(1-a)\sum_{n=0}^{\infty} a^n = \sum_{n=0}^{\infty} a^n - \sum_{n=0}^{\infty} a^{n+1} = \left(1 + a^1 + ... + a^{\infty-1} + a^{\infty}\right) - \left(a^1 + a^2 ... + a^{\infty} + a^{\infty+1}\right)$$

$$(1-a)\sum_{n=0}^{\infty} a^n = 1 - a^{\infty+1} = 1; \; |a| < 1$$

$$\boxed{\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}; \; |a| < 1} \tag{A5}$$

$$(1-a)\sum_{n=0}^{N-1} a^n = \left(1 + a^1 + ... + a^{N-2} + a^{N-1}\right) - \left(a^1 + a^2 ... + a^{N-1} + a^N\right)$$

$$(1-a)\sum_{n=0}^{N-1} a^n = \left(1 - a^N\right)$$

$$\boxed{\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}} \tag{A6}$$

PERIODIC AVERAGES OF A COMPLEX SINUSOID:

$$\frac{1}{T}\int_0^T e^{\frac{j2\pi nt}{T}}\,dt;\quad 0 \le \mathrm{t} \le \mathrm{T}$$

$$= \frac{T}{jT\,2\pi n} e^{\frac{j2\pi nt}{T}}\Bigg|_0^T = \frac{T}{jT\,2\pi n} - \frac{T}{jT\,2\pi n} = 0,\quad n \ne 0$$

$$= \frac{1}{T}\int_0^T e^{\frac{j2\pi 0}{T}}\,dt = \frac{1}{T}\int_0^T 1\,dt = 1;\quad n = 0,\ T,\ 2T, etc.$$

$$\boxed{\frac{1}{T}\int_0^T e^{\frac{j2\pi nt}{T}}\,dt = \delta(n \bmod T)} \tag{A7}$$

$$\sum_{k=0}^{N-1} e^{\frac{j2\pi kn}{N}} = \frac{1 - e^{\frac{j2\pi nN}{N}}}{1 - e^{\frac{j2\pi n}{N}}} = 0;\quad \mathrm{n} \ne 0$$

$$= \sum_{k=0}^{N-1} 1 = N;\quad \mathrm{n} = 0, \mathrm{N}, 2\mathrm{N}, \mathrm{etc.}$$

$$\boxed{\frac{1}{N}\sum_{k=0}^{N-1} e^{\frac{j2\pi kn}{N}} = \delta(n \bmod N)} \tag{A8}$$

AVERAGE POWER OF CONTINUOUS AND DISCRETE COMPLEX SINUSOIDS:

$$\frac{1}{T}\int_0^T \left(Ae^{jk\omega_0 t}\right)\left(Ae^{jk\omega_0 t}\right)^* dt = \frac{1}{N}\sum_{n=0}^{N-1}\left(Ae^{jk\omega_0 n/N}\right)\left(Ae^{jk\omega_0 n/N}\right)^*$$

$$\frac{1}{T}\int_0^T \left(Ae^{jk\omega_0 t}\right)\left(Ae^{-jk\omega_0 t}\right) dt = \frac{1}{N}\sum_{n=0}^{N-1}\left(Ae^{jk\omega_0 n/N}\right)\left(Ae^{-jk\omega_0 n/N}\right)$$

$$\frac{1}{T}\int_0^T \left(A^2 e^{jk\omega_0 t} e^{-jk\omega_0 t}\right) dt = \frac{1}{N}\sum_{n=0}^{N-1}\left(A^2 e^{jk\omega_0 n/N} e^{-jk\omega_0 n/N}\right)$$

$$\frac{1}{T}\int_0^T A^2\,dt = \frac{1}{N}\sum_{n=0}^{N-1} A^2$$

$$\frac{A^2 T}{T} = \frac{A^2 N}{N} = A^2 \tag{A9}$$

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